**Realistic Modeling of the Complex Morphology of CNT-Reinforced Nanocomposite using Immersed Finite Element Simulations**

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Abstract

In this article, we overcome the major difficulties associated with the FE modeling of complex CNT morphologies in polymeric nanocomposites. The difficulties are caused by the continuous mesh model of the traditional FE method, where the CNT and matrix grids must be matched and meshed as one part. These difficulties are typically overcome by the introduction of many unnecessary simplifying assumptions and a much-reduced cell size to reduce the degree of freedom of the problem to enable its treatment. To overcome these unnecessary simplifying assumptions, we adopt an Immersed FE method that allows the CNT and the polymer to be created independently but simulated as a coupled system. In this case, it is assumed that the CNT and the polymeric nanocomposite are fully bonded, and that the equilibrium equations of the system are strictly obeyed. The most important advantage of the newly developed Immersed FE method is that the polymer can always be discretized as a regular grid, irrespective of the complex nature of the CNT network. Our new method made it possible for us to simulate realistic composites containing complex CNT configurations while it is almost impossible by using traditional FE method due to the dramatic difficulties in meshing process and the huge number of elements. Although the polymer and CNT grids are independent in the proposed method, numerical examples demonstrate that the results of the Immersed FE method are very close to the results of the traditional FE method. Finally, the newly developed method is used to study the influence of CNT waviness on the material properties of polymeric nano-composites.

1. **Introduction**

In nanocomposite science, the carbon nanotubes (CNTs) have attracted considerable interest due to their remarkable mechanical and physical properties, such as such as low density [11], high Young’s modulus, strength and fracture strain [1,7,9,19] and high thermal and electrical conductivity [14,26,33]. These superior properties make it an excellent choice to reinforce pure polymer as the multi-functional composite. In the past few decades, intensive attentions are paid to understand the mechanical behavior of the nanocomposite through experimental [20,23,24], analytical [15,17,31] and numerical studies [5,18,24,28].

Numerical simulation is widely used in studying the mechanical properties of nanocomposites and a variety of modeling methods are developed for different scale of interest. The Molecular Dynamics (MD) method is designed for the nano-scale simulation. The constitutive relations of the MD method are derived from the inter-atomic potentials, and the motion of each atom in the system is based on the forces applying on it. MD method is able to see the details of every atom, so it can be used to study the complex phenomena in nano-scale, such as the interfacial properties between CNT and polymer [4,12,30], the effect of the chemical functionalization on carbon atoms [27,35] and the influence of the defect in the CNT [8,25]. However, even with the usage of the most advanced supercomputers, only about atoms, namely about 1000 atoms in each direction, can be simulated [10]. So it is unrealistic to use MD method in a larger scale.

To enlarge the simulation scale, the traditional continuum model is introduced which assumes the material to be homogeneous and neglects the details of atomic structures. Although the traditional continuum models cannot accurately describe the phenomena in nano-scale, it is able to provide valuable information to study the influences of the micro-scale parameters on the material properties of CNT-reinforced composites, such as the CNT curvature [2,3,22], aspect ratio and volume fraction [13],

Finite Element (FE) method is the most commonly used technique to simulate nanocomposites in micro-scale. However due to the extreme complexity of the CNT net morphology, it is almost impossible to create a FE model for a realistic composite. The major challenge comes from the meshing process, because the traditional FE method requires the CNT and polymer grids to be matched with each other and meshed as one part. The difficulty increases significantly with the increment of CNT number and morphological complexity. Lusti and Gusev created the traditional FE model of straight CNT-reinforced composites by using a complicated three dimensional mesh generator [39]. Fig.1 shows one of their FE models. A lot of efforts are made to create it but there are still many highly distorted cells which may decrease the accuracy of the results. Moreover, the number of freedom of degree is also very huge. The volume fraction of CNT in Fig.1 is only 0.5%, but the composite is meshed into tetrahedral elements. Alian and Meguid also used the traditional FE model to study the mechanical-electrical behavior of composites reinforced by straight CNTs [6], but due to the difficulties in the meshing process, the CNT volume fraction is only 0.3% in their research. The problems in the meshing process can be even more severe when consider the waviness and agglomeration in the realistic composites.

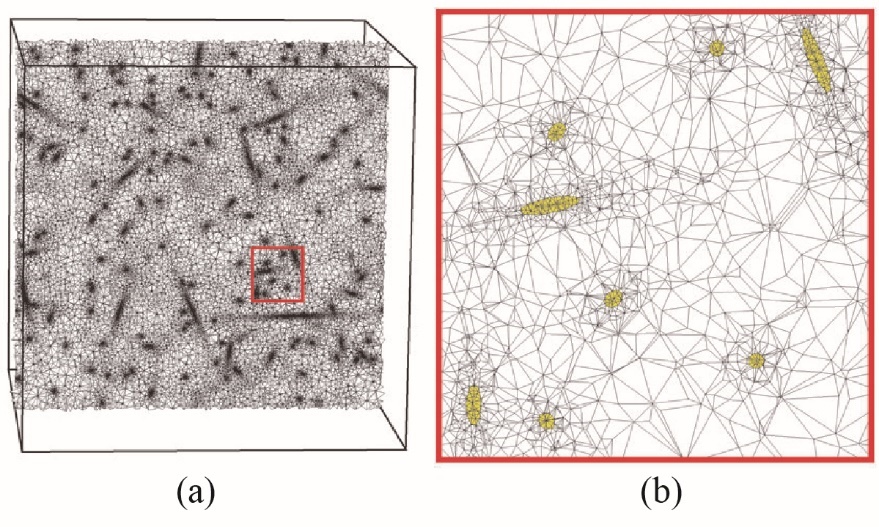


Figure.1: The traditional FE model of a composite with straight CNT. (b) is the magnification of the red framed section shown in (a).

To avoid the complicated meshing process, many assumptions are made to overcome the difficulties. The first kind of assumption is to reduce the number of CNT and the morphological complexity to reduce the freedom of degree. For instance, Fisher studied the influence of the waviness by FE models which contain only a single nanotube [22]. Chen did research about the mechanical response of composites with regular CNT distribution [36]. The other kind of assumption is to make the use of the effective material properties. Mori-Tanaka model [29] is used by Rafiee to avoid the explicit meshing of CNT grids, but the details of the CNT deformation are also ignored [21]. The Mori–Tanaka model was also used by Alian and Meguid in their studies about CNT waviness and alignment. More recently, the “embedded element technique” [22,40] is used to simplify the meshing process. The CNTs are meshed as beam elements and their freedom degrees are embedded into the polymer by modifying the stiffness matrix of each polymer element. However, only kinematic relation between CNT and polymer is considered in this method while the more important issue, namely the interacting forces in between, is not accounted for. In summary, unnecessary simplifying assumptions are introduced to overcome the difficulties in the meshing process, and they will also decrease the accuracy of the results.

In this study, we adopt an Immersed FE method that allows the CNT and the polymer to be created independently but simulated as a coupled system. It means that the polymer can always be discretized as a regular grid, irrespective of the complex nature of the CNT network. Our new method made it possible for us to simulate realistic composites containing complex CNT configurations while it is almost impossible by using traditional FE method. The CNT net and polymer are coupled according to a basic “fully bond” assumption. Rather than the “embedded element technique” which just embeds the CNT freedom degree, the equilibrium equation of each CNT is embedded into the equilibrium equation of the polymer according to the Newton's third law. Therefore, both kinematic and mechanic relations between CNT and polymer are considered in the Immersed FE method. Since no unnecessary assumption is introduced, numerical examples demonstrate that the results of Immersed FE method are very close to the results of the traditional FE method.

It should be mentioned that the Immersed FE method is not only suitable to simulate the nanotube-reinforced composites, it can also be used to simulate the mechanical behavior of particle-reinforced composites in the same way.

The remaining of this paper is organized as follow. In Section 2, the basic idea, assumption, governing equation and the solution scheme of the Immersed Finite Element are introduced. Subsequently in Section 3, two numerical examples are presented to validate the results from the proposed method. Subsequently, the Immersed FE method is used to study the influence of CNT waviness on the material properties of nano-composites. Finally, the conclusion is summarized in Section 4.

1. Immersed Finite Element Method

Fig.2 is the flowchart of using the Immersed FE method to simulate the nanocomposite with complex CNT configurations. Due to the extremely complex CNT network morphology in Fig.2(a), it is very difficult to create the traditional FE model where the CNT and polymer grids are matched and meshed as one part. But in the Immersed FE method, the polymer grid could just be meshed as a regular grid shown in Fig.2(b) and immerse the CNT network into the polymer grid as shown in Fig.2(c). It is easy to create the composite model without the consideration about the complex CNT distribution and thus, make it possible for us to simulate realistic composites containing complex CNT configurations

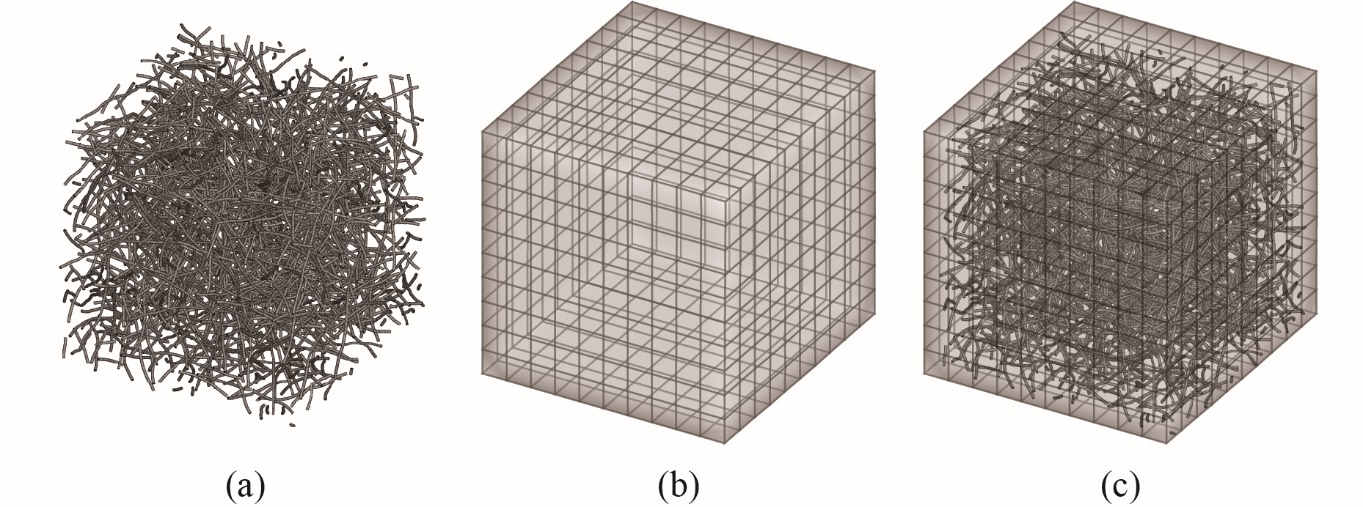


Figure.2: The flowchart of the Immersed FE method.

The following of this section will introduce the basic idea, assumptions, governing equation and the solution scheme of the Immersed Finite Element.

* 1. Basic assumption and the governing equation

The polymer and the CNTs are both meshed with 8 node hexahedral elements in the proposed Immersed FE method, but they are not matched. The basic assumption is the polymer and CNTs are fully bonded, namely the displacement of the CNT surface nodes equals to the local polymer displacement. This assumption is widely used in continuum model [2,6,13,36] although they are not perfectly bonded in reality and may influence to effective material properties [16]. But it is also reasonable for small deformation where the stress in CNT can fully transferred to the polymer [32].

In the Immersed FE method, the CNTs' grid are not matched with the polymer grid, so the displacement of the CNT surface nodes should be gathered from the surrounding polymer nodes. Once the surface displacement of the *i* th CNT, namely is determined, the internal displacement and the reacting force can be obtained by solving the equilibrium equation

⑴

is the force that the polymer applies on , so the force that applies on polymer is according to the Newton's third law. It should be mentioned that this force is not exactly applied on the polymer nodes, so it should be scattered to the surrounding polymer nodes. Since is calculated is from the polymer displacement , the force that applies on the polymer should also be a function of and denote it as .

Then, the equilibrium equation of polymer is

⑵

Eq.⑵ is the governing equation of the Immersed FE method and the whole system can be determined after solving this equation.

* 1. Properties of the governing equation

We will prove that Eq.⑵ leads to a linear equation with positive definite coefficient matrix in the following.

Suppose the number of freedom degree in the polymer grid is and the number of the boundary freedom degree in is . For the *j* th boundary freedom degree in , its displacement is gathered from the surrounding polymer nodes, namely

⑶

where is the contribution of the *k* th polymer freedom degree to the *j* th boundary freedom degree of in the displacement gathering. Eq.⑶ can be rewritten as

⑷

where is the displacement gathering matrix with dimension .

The solution of Eq.⑴ is

⑸

where must be a positive definite matrix due the conservation of energy. Therefore, the force that applies on polymer is

⑹

and it should be scattered to the surrounding polymer nodes, namely

⑺

where is the force scattering matrix with dimension .

Substituting Eq.⑷ into Eq.⑺, we have

⑻

In this paper, the isoparametric interpolation of the 8-node hexahedral element is used to gather displacement and scatter force. So , and Eq.[(8)](#eq_8) can be rewritten as

⑼

where is a positive semi-definite matrix.

Finally, Substituting Eq.⑼ into Eq.⑵ leads to

**⑽**

is the total stiffness matrix of the nanocomposite which contains the original stiffness from the polymer, namely ***K***, and the reinforced stiffness from CNTs, namely ***S***.

Strictly speaking, ***K*** should not be equal to the stiffness matrix of the pure polymer . This is because the CNTs in the composite occupy some space of the polymer and thus, decrease the stiffness of the polymer. Nevertheless, since the volume fraction of the nanocomposite is very low (generally less than 5%), and the stiffness of CNT is much higher than that of the polymer (about 300 times), the magnitude of should be much less than the magnitude of ***S***. So in this paper, is used in Eq.**⑽** for the sake of simplicity.

Dividing into the active freedom degrees and the constraint freedom degrees, Eq.**⑽** changes to

⑾

and the Finally equation is

⑿

It should be mentioned that is a positive definite matrix and is a positive semi-definite matrix, so the coefficient matrix of Eq.⑿ is positive definite.

* 1. Solution scheme

Although Eq.⑿ is a linear equation, it is almost impossible to calculate the coefficient matrix directly because the expression of and can be very complicated. However, can be easily obtained by solving Eq.⑴ for each CNT and summing over all . Therefore, the CG (Conjugate Gradient) iteration can be used to solve Eq.⑿.

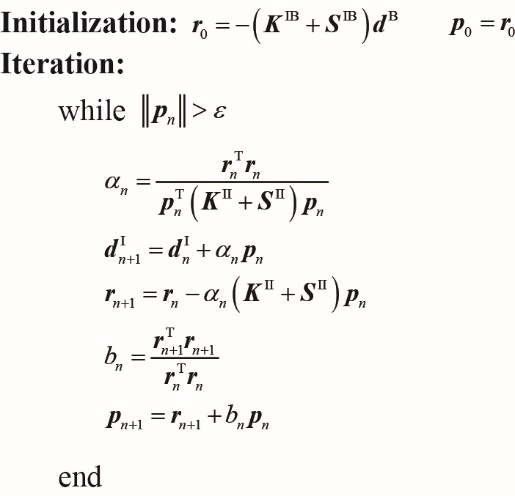
The flowchart of the CG iteration is shown in Algorithm.1 where and are known while and are unknown variables. Note that

⒀

so equals to the components of related to the active freedom degree. Similarly,

⒁

so equals to the components of related to the active freedom degree subtracting .



Algorithm 1: The flowchart of the CG iteration.

1. Validation and numerical results

In this section, two numerical tests are presented at first for the validation of the method proposed in this paper, namely to demonstrate that the results from the Immersed FE method are close to the results from the traditional FE method. the Immersed FE method is used to study the influence of CNT waviness on the material properties of nano-composites.

* 1. One CNT validation

In this example, one SWCNT selected to be (5,5) armchair of radius and length is put into a polymer. The size of polymer is and the CNT is displaced in the center of the polymer as shown in Fig.3. The material properties of the CNT and polymer come from MD simulation, namely , and , . The boundary condition is at face , at face , at face z, and at face .

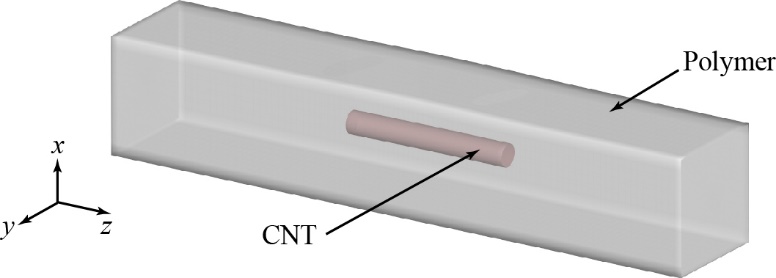


Figure.3: The model of the one CNT validation.

This problem is simulated by both the traditional FE method and the Immersed FE method proposed in this paper. The local view of the traditional FE model near the CNT is shown in Fig.4 and the CNT and polymer meshes are matched. The number of cells in the traditional FEM model is 168300. In the Immersed FE model, the CNT grid is the same with that in the traditional FEM while the polymer is meshed as a regular grid with cells. The number of cells in these two models is close, so they have a similar grid resolution.

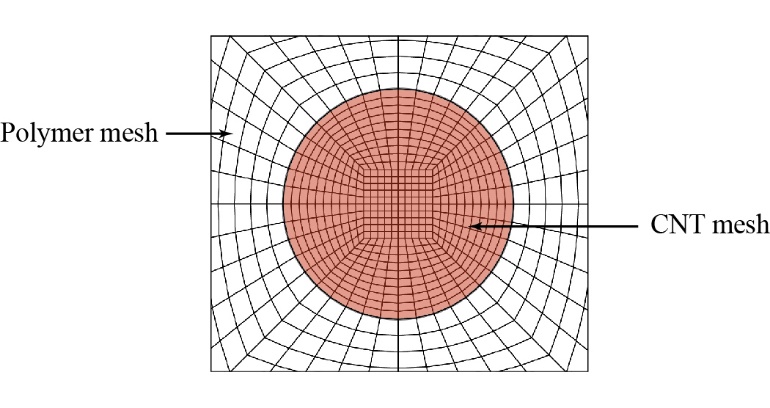


Figure.4: Mesh-matched FEM model

The contour of the CNT are compared in Fig.5. The distribution in Fig.5(a) and (b) are very similar in most place while the stress at the endpoints of the CNT are overestimated. The overestimation may occur if one or more CNT cells at the endpoint are fully inside a polymer cell, and it can be eliminated by using a refined polymer grid or a coarser CNT grid. Fig.5(c) shows the contour if the polymer is meshed in , and the stress overestimation is almost disappeared.

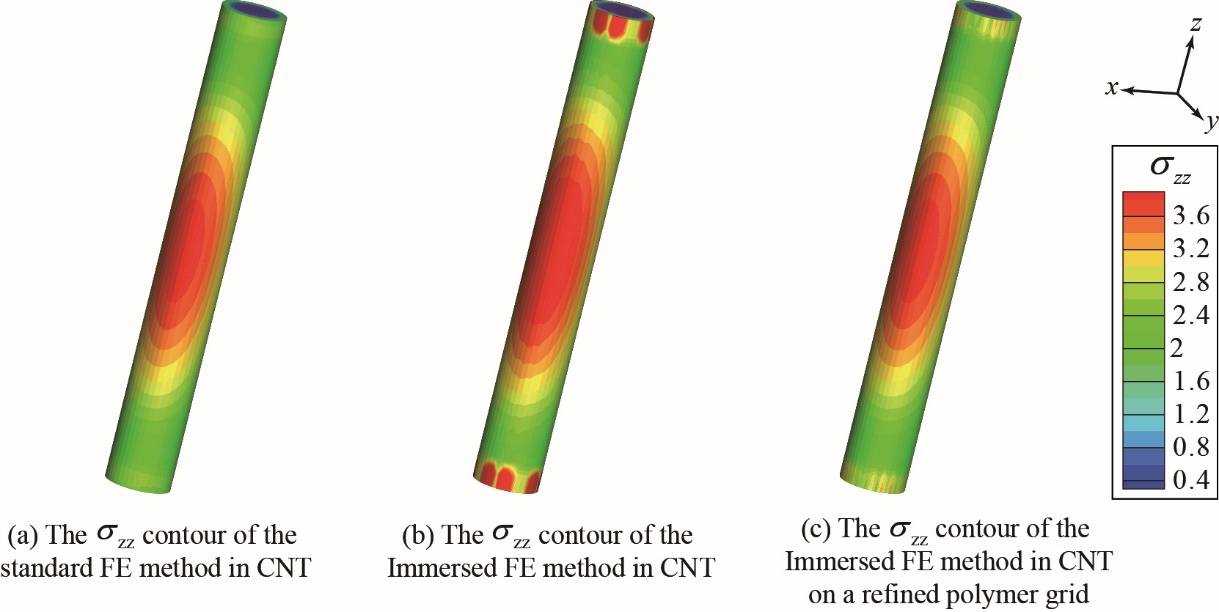


Figure.5: The comparison of the in CNT.

Fig.6 presents the polymer comparison near the CNT, and the stress distribution are also very similar in most place. The degree of the stress concentration phenomenon near the CNT's endpoints is underestimated due to the non-matched polymer and CNT grid, and it can also be eliminated by using a refined polymer grid as shown in Fig.6(c). It should be mentioned that the underestimation is localized because the equilibrium equation is an elliptic equation. Therefore, it would not produce obvious error when evaluating the effective material properties of the nanocomposite. Fig.7 shows the contour at the polymer boundary, and the results from the traditional and Immersed FE method are almost the same.

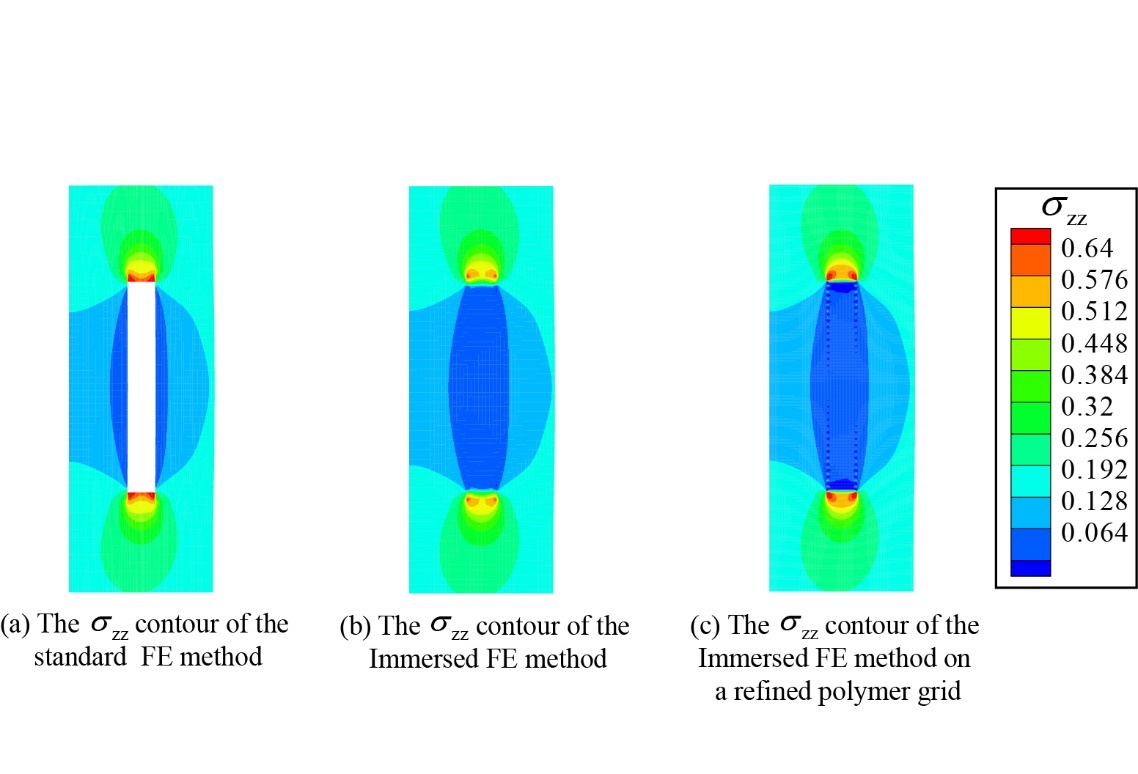


Figure.6: The comparison of in polymer near the CNT.

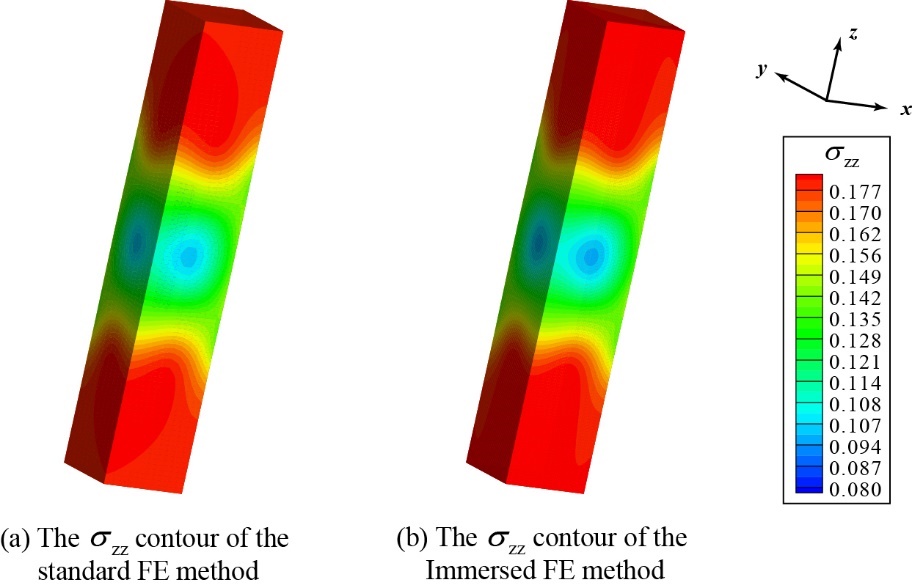


Figure.7: The comparison of in polymer near the boundary.

* 1. CNT nanocomposite validation

In this example, the Immersed and traditional FE models of CNT-reinforced composites with different volume fractions are created. The material properties of the CNT and polymer are same with the example in Section 3.1 and the length of the representative volume element (RVE) is 70nm. Eight different composites are created with CNT volume fraction *f* varying from 0.025% to 0.2%. Fig.8 shows the CNT distribution at *f*=0.05%, 0.1% and 0.2%.

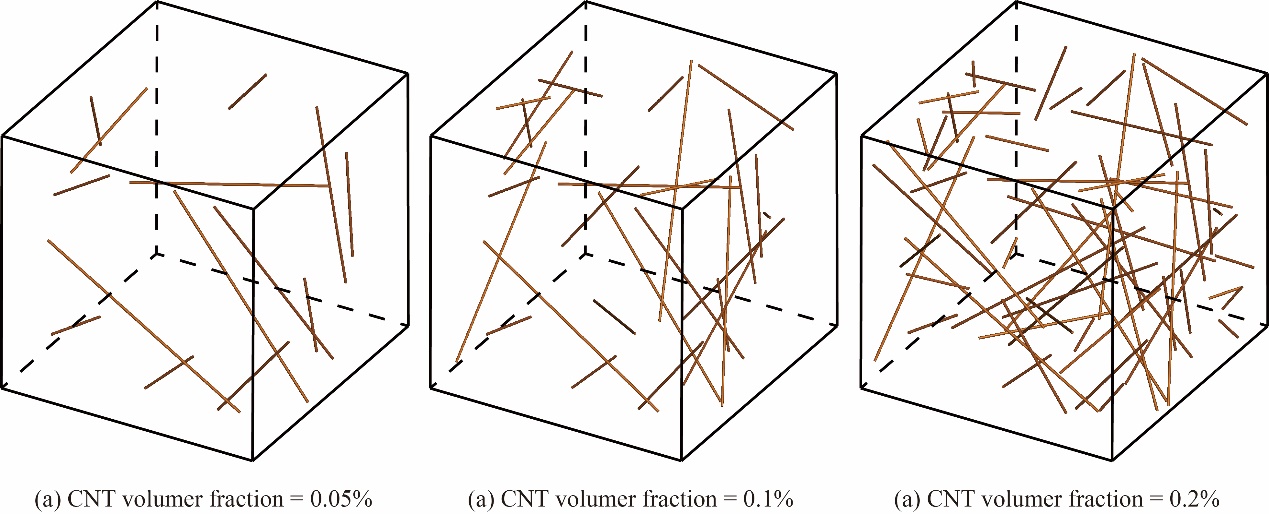


Figure.8: The CNT networks with different volume fractions.

The , and of these composites are calculated by both Immersed and traditional FE method, and the average Youngs modulus from the two methods are compared. The traditional FE models are discretized by 4-node tetrahedron elements. In the Immersed FE model, the size of the CNT element is the same with the traditional FE model while the polymer is discretized as regular grid with different resolutions. The polymer grid resolution is defined as

⒂

where is the diameter of the CNT and is the length of the polymer element.

Fig.9 shows the average Youngs modulus from the Immersed and traditional FE method. The results of Immersed FE method are dependent on the polymer grid resolution, and they can gradually close to the results of traditional FE by using higher grid resolution. At resolution , the average Youngs modulus from the Immersed and traditional FE method are almost coincident. Fig.10 shows curve about the average relative error of between the Immersed FE and traditional FE method. It also clearly indicates that the results of the Immersed FE method can be more accurate by using higher polymer grid resolution.

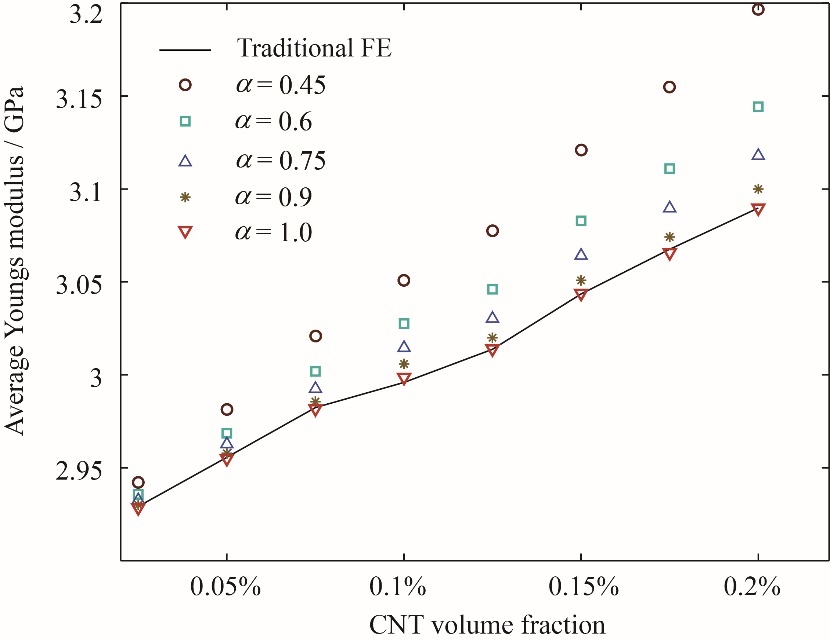


Figure.9: The comparison of average Youngs modulus between the traditional FE

method and the Immersed FE method with different polymer grid resolution.

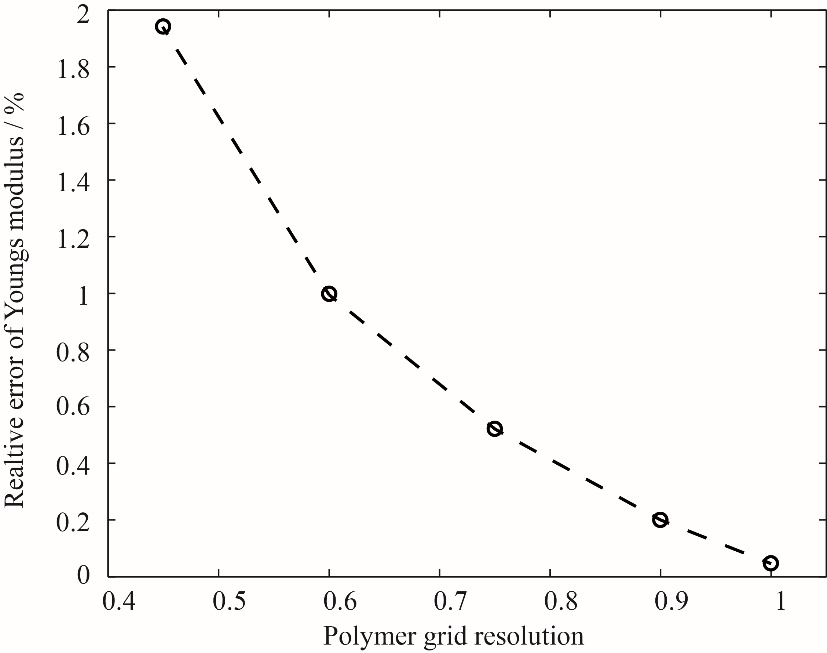


Figure.10: Variation of relative error of Youngs modulus versus polymer grid resolution.

To demonstrate the advantages of the Immersed FE method, Fig.11 shows variation of the increment of element number in Immersed FE and traditional FE models versus the increment of the CNT volume fraction.

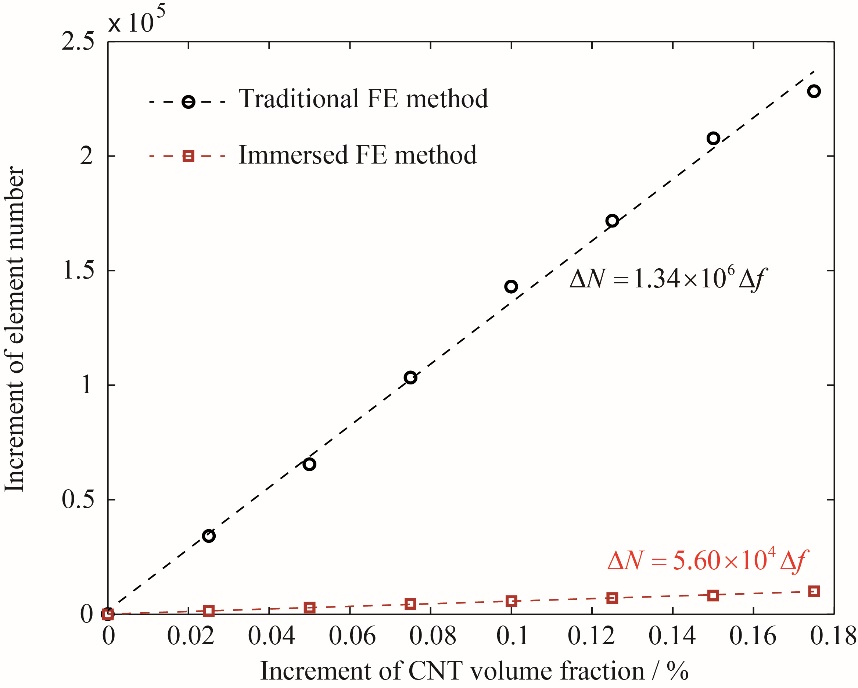


Figure.11: The relationship between the increment of element number and the increment of CNT volume fraction.

The results indicate that the slope of traditional FE model is about 24 times larger than the slope of Immersed FE model. In fact, once the polymer grid resolution of the Immersed FE model is fixed, the increment of the total element number only equals to the increment of CNT element number. However, in the traditional FE model, the polymer should be re-meshed when adding new CNTs and a lot of new polymer elements are created around each CNT. It leads to the significantly higher slope in Fig.11. Besides the huge number of element, the difficulties in meshing process also increases dramatically at high volume fraction. The Immersed FE method does not face these difficulties but can provide the similar results of the traditional FE method, so it is an effective approach to simulate the realistic complex morphology of CNT-reinforced nanocomposite.

* 1. Influence of CNT waviness on material properties of nano-composites

To validate the capability of the proposed method in solving complex problems, the Immersed FE method is used to study the effect of CNT volume fraction and waviness on the Young’s modulus of nano-composites. The CNT is selected to be Single-Walled (5,5) armchair with *r* = 0.67 nm and *l* = 67nm. These CNTs are uniformly and fully dispersed in a brick RVE with size *L* = 67nm. “Fully dispersed” means that the minimal distance of any two CNTs in the network must larger than where is the distance of Van Der Waals force [6]. The combination of 3 volume fractions and 4 curvatures are studied. At each combination , 3 sets of random CNT network are created by Monte Carlo method. Periodic condition is applied at the RVE boundary. Fig.12 shows 3 CNT net samples with and different curvatures. The CNT morphologies are very close to the realistic SEM image [3] in Fig.11(d). It is very difficult or even impossible to create the traditional FE models for these composites, but it can be easily simulated by using the Immersed FE method proposed in this work. The polymer grid resolution is for all cases.

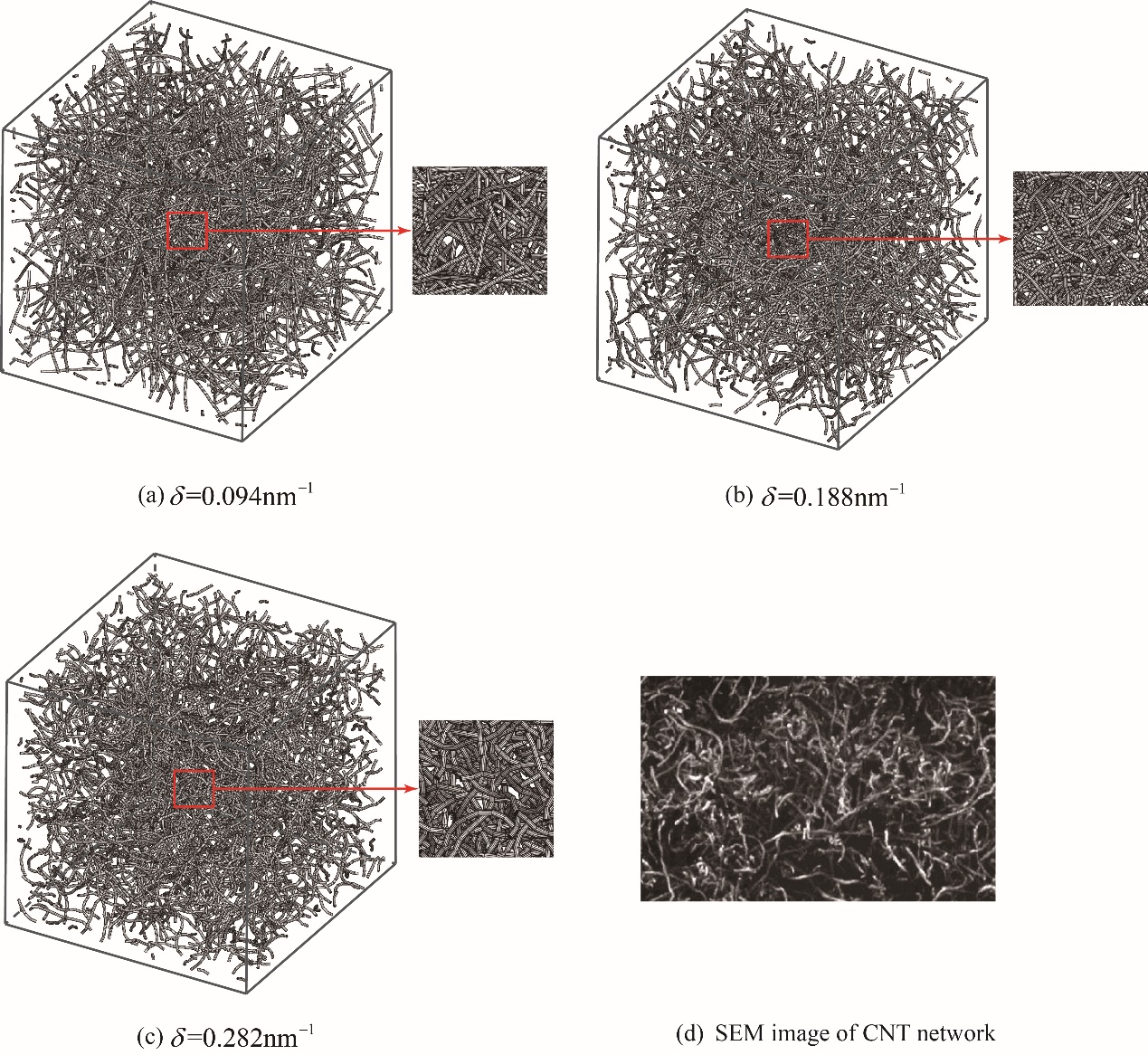


Figure.12: The curved CNT nets with different curvatures

Fig.13 and Fig.14 show the variation of Young’s modulus of the nanocomposites versus the CNT curvature and volume fraction. The effective Young’s modulus at each combination is the average of the 3 samples. Fig.12 indicates that the effective Young’s modulus decrease with increasing CNT curvature and this phenomenon is also observed in many other studies [2,3,22,37]. The numerical results are also compared with the experimental results of CNT–epoxy composites [38] and the result at matches well with the experimental data. The good agreement between the reported experimental data and the numerical results also validates the capability of the proposed method in solving complex problems.

In addition, Fig.13 demonstrates that the decrement of effective Young’s modulus is almost linear from to and the slopes for are , and , respectively. It demonstrates that the reduction of Young’s modulus becomes more tangible with higher CNT volume fraction and this conclusion is similar with the study by Roham [21].

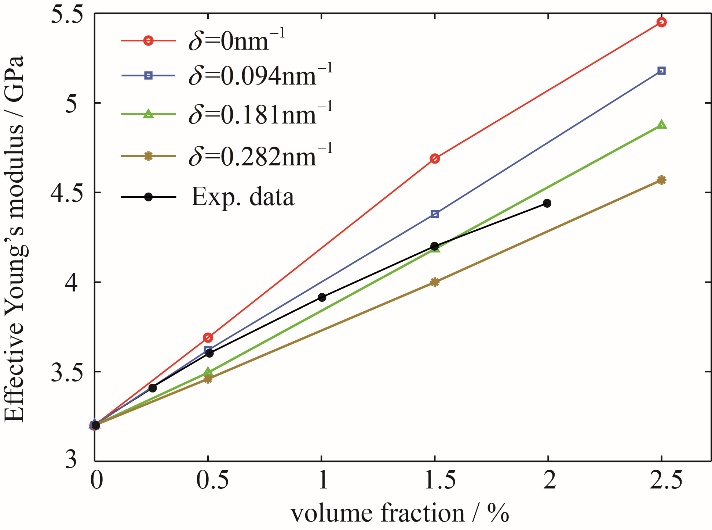


Figure.13: Variation of effective Young’s modulus of the nanocomposites with different curvatures versus volume fraction

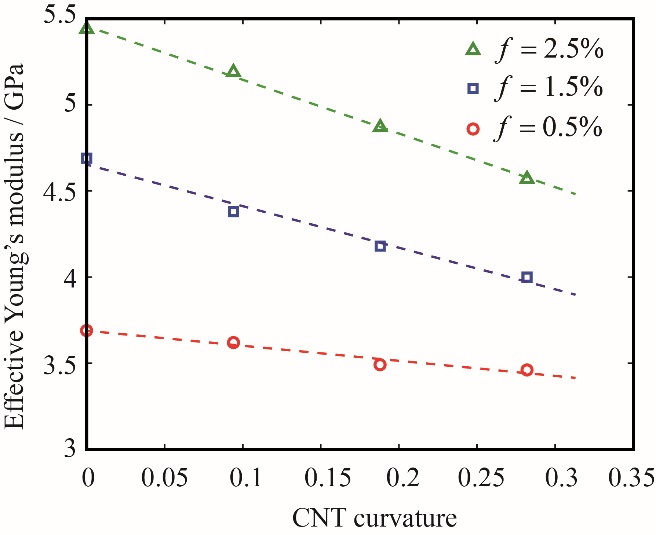


Figure.14 Variation of effective Young’s modulus of the nanocomposites with different volume fractions versus curvature

1. Conclusion

In this study, we propose an Immersed Finite Element method to simulate the mechanical behavior of CNT-reinforced polymer. In our proposed method, the CNT network and the polymer can be created independently but simulated as a coupled system. The composite model can be easily obtained without the need to consider the complex CNT distribution, so it is possible for us to simulate realistic composites containing complicated CNT configurations. When coupling the independent CNT and polymer grids, both kinematic and mechanic relations in between are considered by embedding the equilibrium equation of each CNT into the equilibrium equation of polymer. Since no additional unnecessary assumption is introduced, the results of the Immersed FE method are very close to the traditional FE method. The proposed method is used to study the influence of CNT waviness on the Young’s modulus of nanocomposites. The morphology of the CNT network in our study is very similar to the reality and the numerical results indicate that the CNT waviness plays a negative role in the reinforcement. The numerical results show good agreements with the experimental data and the conclusions of previous studies, thus validating the capability of the Immersed FE method in simulating the realistic composites with complex morphology.

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